On the Cusp around Central Black Holes in Luminous Elliptical Galaxies

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ABSTRACT

In this letter, we show that a massive black hole (MBH) which falls into the center of a galaxy in dynamical timescale leaves a weak cusp ($\rho \propto r^{-1/2}$) around it, which is in good agreement with the recent observations of luminous ellipticals by *Hubble Space Telescope*. Such event is a natural outcome of merging of two galaxies which have central MBHs. This is the only known mechanism to form weak cusps in luminous ellipticals. Therefore, the existence of the weak cusps indicates the central BHs of luminous ellipticals have fallen to the center from outside, most likely during a major merger event.

Subject headings: galaxies: elliptical and lenticular, cD — galaxies: kinematics and dynamics — galaxies: nuclei — galaxies: structure

1. Introduction

Recent observations of elliptical galaxies by *Hubble Space Telescope* (*HST*) (Lauer et al. 1995; Byun et al. 1996; Gebhardt et al. 1996; Faber et al. 1997; Kormendy et al. 1996) have brought us some challenging problems as well as new knowledges on the structure of the central regions of elliptical galaxies. First, these observations showed that no elliptical has "core", in which the surface brightness or the luminosity density profiles would be flat. The central regions formerly regarded as "cores" proved themselves to be power-law cusps in all observed galaxies. Second, the distribution of the slopes of these cusps seem to be bimodal, one is the group of "core" galaxies with weak cusps ($\rho \propto r^{-\alpha}, 0.4 \lesssim \alpha \lesssim 0.8$) and the other is the group of "power-law" galaxies with steep cusps ($\rho \propto r^{-\alpha}, \alpha \sim 2$). In addition, the slopes of the cusps have correlation with the brightness of the galaxies, so that bright galaxies tend to have weak cusps. Carollo et al. (1997) claimed that the distribution of the slope is rather continuous in their sample. However, their sample is limited in the range of absolute magnitude, so it is not clear whether their result is real or due to selection.

The models of cuspy stellar systems previously studied are classified into two categories — models with and without MBH. However, neither can explain the origin of the weak cusps in luminous elliptical galaxies. It was shown that the cusp around BH would have the profile $\rho \propto r^{-7/4}$ when the evolution is driven by the thermal relaxation (Bahcall & Wolf 1976; Shapiro & Marchant 1978; Cohn & Kulsrud 1978; Marchant & Shapiro 1979; Marchant & Shapiro 1980) and $\rho \propto r^{-3/2}$ (Young 1980; Merritt & Quinlan 1998) or rather steeper (Quinlan, Hernquist, & Sigurdsson 1995) when the central BH grows adiabatically. Hierarchical clustering in CDM cosmogony (Navarro, Frenk, & White 1996; Fukushige & Makino 1997) or dissipationless collapse (Hozumi, Burkert, & Fujiwara 1999) might form relatively shallow cusps, but they are still significantly steeper than the observed weak cusps.

Makino & Ebisuzaki (1996, hereafter ME96) showed that a weak cusp ($\rho \propto r^{-\alpha}, \alpha \lesssim 1$) is formed through the merging of two galaxies which have central BHs. They also found that the ratio between the size of the weak cusp region and the half-mass radius of the merger remnant, $r_{\rm c}/r_{\rm h}$, is proportional to the ratio between the BH mass and the galaxy mass $M_{\rm BH}/M_{\rm g}$. This result is in good agreement with observations (Gebhardt et al. 1996). However, they did not discuss why the cusp was formed. Quinlan & Hernquist (1997) and Makino (1997) showed that the BH binary in galactic center ejects many stars as it hardens and this process can explain the weak cusps in large ellipticals, but they made no quantitative prediction about cusp slopes.

In our previous work (Nakano & Makino 1999, hereafter NM99), we investigated the dynamical reaction of a galaxy to a BH which falls to the center, in order to clarify the formation mechanism of the cusp. We found that when the massive BH falls to the galaxy center, the stars are heated up by the BH and the weak cusp ($\rho \propto r^{-1/2}$) is formed (Figure 1). This result is independent of the initial orbital angular momentum and the mass of BH. Thus, we can conclude that when a BH (or BHs) falls from outside to the center of a galaxy, a central weak cusp is always formed. However, it was not at all clear why the cusp is formed.

In this letter, we present the theoretical explanation of the formation mechanism of the weak cusp.

2. Why the Cusp Tends to $r^{-1/2}$?

In the results of our N-body simulation, we found an important feature of the energy distribution function $N(\mathcal{E})$ profiles, shown in Figure 2. The system of units we use is the standard unit (Heggie & Mathieu 1986), in which the total mass of a galaxy $M_{\rm gal} = 1$, the gravitational constant $G = 1$ and the total energy of the galaxy $E_{\rm tot} = -1/4$. In this unit, the virial radius of the galaxy is scaled to unity and the half-mass crossing time is $2\sqrt{2}$.

In Fig. 2, $N(\mathcal{E})$ is practically unchanged from that of the initial King model for the runs in which the BH is initially placed off-center. Thus, there is no star with $\mathcal{E} > 3.3$. This is in sharp contrast with $N(\mathcal{E})$ for the run in which the BH is initially placed on-center (dash-dot-dash line). In this case, $N(\mathcal{E})$ has a long tail to $\mathcal{E} \to \infty$. By taking such depletion of tightly bound stars into account, we can explain the existence of the weak cusp as follows.

The density $\rho(r)$ is derived from the distribution function $f(\mathcal{E})$ as

$$\rho(r) = 4\pi \int_0^{\Psi(r)} f(\mathcal{E}) \sqrt{2\left[\Psi(r) - \mathcal{E}\right]} d\mathcal{E}, \qquad (1)$$

where $\Psi(r)$ is the depth of the gravitational poten-

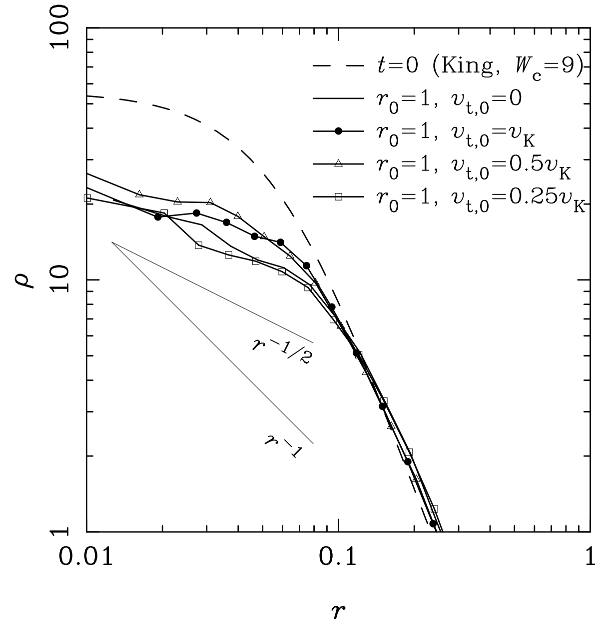

Fig. 1.— Density profiles obtained by N-body simulations of BH "fall-in" (Nakano & Makino 1999). In these runs, the BH mass $M_{\rm BH} = 1/24 M_{\rm gal}$. BH is placed at distance of $r_0 = 1$ from the center with tangential velocity $v_{\rm t,0}$. Note that $v_{\rm K}$ denotes the initial Kepler velocity. In all runs, initial radial velocity of BH is zero.

tial at distance r from the center and $\mathcal{E}$ is the specific binding energy. Here we assume the galaxy is spherically symmetric and the velocity distribution is isotropic. When the galaxy has the central BH, $\Psi(r)$ near the BH can be approximated as $\Psi(r) \sim GM_{\rm BH}/r$, and $\Psi(r)$ diverges when r goes to zero. As shown in Fig.2, $N(\mathcal{E})$ of the galaxy after BH fell down to the center vanishes at some finite value of the binding energy. We denote this limit of binding energy as $\mathcal{E}_0$. Thus $f(\mathcal{E})$ also vanishes at $\mathcal{E}_0$, since $f(\mathcal{E}) = N(\mathcal{E})/A(\mathcal{E})$ where $A(\mathcal{E})$ is the area of hypersurface in phase space with energy $\mathcal{E}$ (Binney & Tremaine 1987). Thus, if $\Psi(r) > \mathcal{E}_0$, eq. (1) can be rewritten as

$$\rho(r) = 4\pi \int_0^{\mathcal{E}_0} f(\mathcal{E}) \sqrt{2\left[\Psi(r) - \mathcal{E}\right]} d\mathcal{E}. \qquad (2)$$

In the region $\Psi(r) > \mathcal{E}_0$, Eq. (2) can be expanded as

$$\begin{aligned} \rho(r) &= 4\pi \int_0^{\mathcal{E}_0} f(\mathcal{E}) \sqrt{2\left[\Psi(r) - \mathcal{E}\right]} d\mathcal{E} \\ &= 4\sqrt{2}\pi \sqrt{\Psi(r)} \\ &\quad \times \int_0^{\mathcal{E}_0} f(\mathcal{E}) \left[1 - \frac{1}{2}\frac{\mathcal{E}}{\Psi(r)} + O\left(\left[\frac{\mathcal{E}}{\Psi(r)}\right]^2\right)\right] d\mathcal{E} \\ &\propto \sqrt{\Psi(r)} \sim \sqrt{\frac{GM_{\rm BH}}{r}}. \end{aligned} \qquad (3)$$

Therefore, in the central region where $\Psi(r) \gg \mathcal{E}$, $\rho(r)$ is proportional to $r^{-1/2}$.

Note that our theory is related to, but not the same as, the theory for $r^{-1/2}$ cusp by Zel'dovich & Novikov (1971) and Peebles (1972). They showed that if a massive object is placed in a stellar system of uniform density (such as very large core), there will be small cusp with $\rho \propto \sqrt{1 + (GM_{\rm BH} m / r E_\infty)}$, where E_∞ is the energy of a field star. In their theory, this cusp is due to gravitational focusing of stars of uniform background. In our theory, the cusp is also due to stars which are not bound to BH, but we showed that uniform background is not necessary. Our explanation needs only one assumption: $f(\mathcal{E})$ vanishes at a certain finite energy. This assumption is quite natural for the remnant of the merging of galaxies with MBHs, since MBHs would heat up the stars through the back reaction of the dynamical friction from the stars to the falling MBHs.

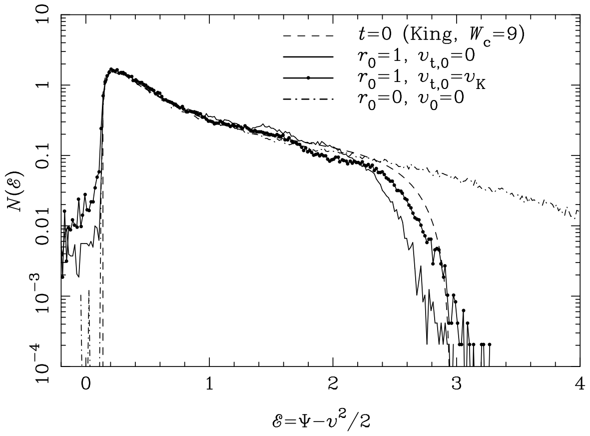

Fig. 2.— The profiles of the binding energy distribution $N(\mathcal{E})$ of the galaxy after the BH settled in the center, obtained by our N-body calculation (Nakano & Makino 1999), where $\mathcal{E}$ is the binding energy, $\mathcal{E} = \Psi - v^2/2$, and Ψ is the potential energy defined to be minus the conventional gravitational potential. For all runs, the initial galaxy model is an isotropic King model with non-dimensional central potential $W_c = 9$. The mass of the BH, $M_{\rm BH}$, is 1/24 of the mass of the galaxy.

3. Self-consistent Model of Weak Cusp

Figure 3a shows the self-consistent solutions of the density profiles of the galaxy with the central BH, for three different forms of $N(\mathcal{E})$; a King model with $W_c = 9$ (solid), a constant $N(\mathcal{E})$ (dot-dashed) and exponential with cutoff $N(\mathcal{E}) \propto (e^{\mathcal{E}_0 - \mathcal{E}} - 1)$ (dotted), where $\mathcal{E}_0$ is cutoff energy. The mass of BH is $1/24 M_{\rm gal}$. We used the iterative method introduced by Binney (1982) to obtain these self-consistent solutions. Figure 3b is the same result as Fig.3a but using the Hernquist model as initial guess of iteration. In all cases, the steep outer slope switches to the weak cusp with $\rho \propto r^{-1/2}$ at around $r = 0.1$. We can clearly see that the difference in the functional form of $N(\mathcal{E})$ does not affect the slope of the central cusp. This result is in good agreement with our analytical explanation described above.

Figure 4 shows the solutions of the density profiles with different BH masses. The size of the weak cusp region (or the so-called "break radius") is larger for larger BH mass, but the slope of the weak cusp remains unchanged.

The relation between BH mass and size of weak

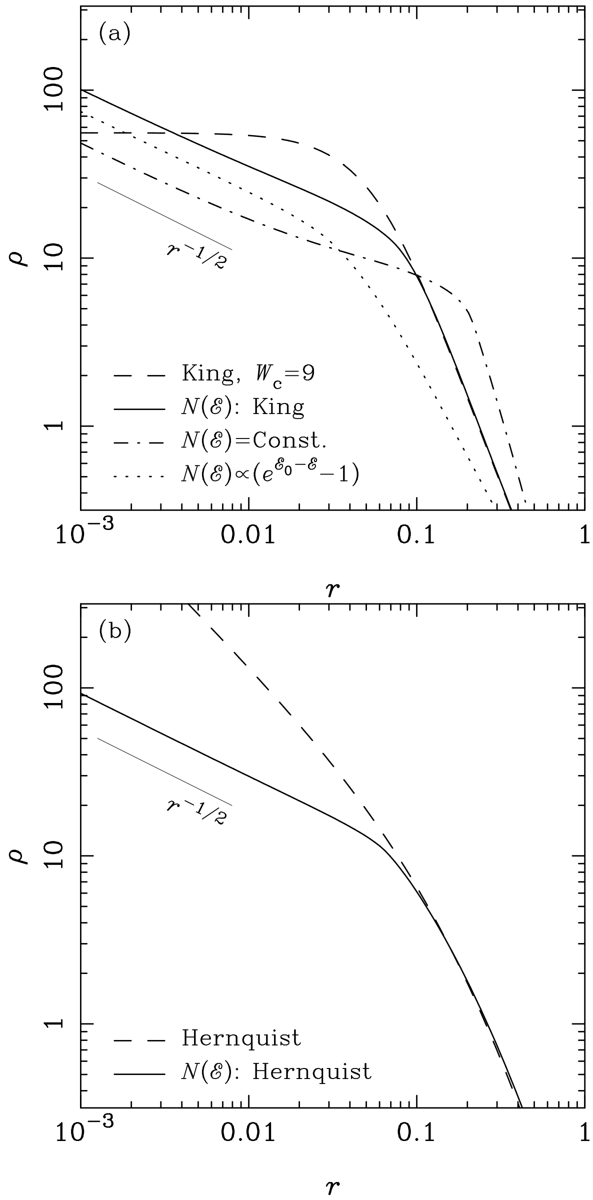

Fig. 3.— (a) Self-consistent solutions of the density profile for various $N(\mathcal{E})$, obtained by the iterative method (Binney 1982). (b) Same results as (a) but using the Hernquist model as initial galaxy model.

cusp region (hereafter simply term it "core") can be understood as follows. Suppose that the inner region of initial power-law density profile $\rho = K_1 r^p$ is transformed by sinking BH to a shallow cusp $\rho = K_2 r^q$ with radius r_c, where $-3 < p < q < 0$. Strictly speaking, a King model has a small flat core, but we can neglect its contribution in the following discussion. Roughly speaking, the total mass of the region affected by the BH must be about the same as that of BH. Thus we have

$$M_{\rm BH} + \int_0^{r_c} K_2 r^q \cdot 4\pi r^2 dr \simeq \int_0^{r_c} K_1 r^p \cdot 4\pi r^2 dr. \quad (4)$$

Using $K_2 r_c^q = K_1 r_c^p$, we obtain

$$r_c \propto {M_{\rm BH}}^{1/(p+3)}. \quad (5)$$

If the initial profile is isothermal ($p = -2$), the core radius would be proportional to BH mass.

Such correlation between BH mass and core size is consistent with the observations of ellipticals, which indicate that there are linear correlations between the core radius and the effective radius (Faber et al. 1997) and between the central BH mass and total mass (Magorrian et al. 1998).

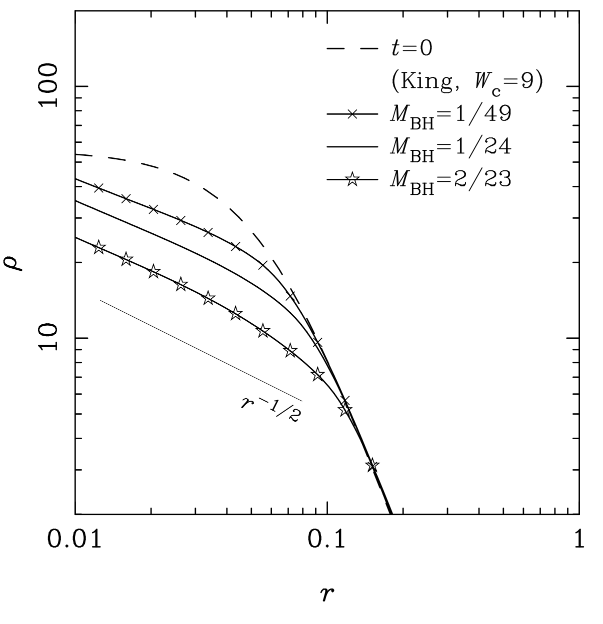

Fig. 4.— Self-consistent density profiles with different BH masses. The initial model is the King model with $W_c = 9$.

4. Summary

In this paper, we showed that the density cusp around MBH has the slope of $-1/2$ if the MBH has "fallen" to the center in dynamical timescale. Thus, we now understood why such weak cusps were formed in numerical simulations of merging with central BHs (Makino & Ebisuzaki 1996) or simulation of sinking BHs (Nakano & Makino 1999).

This is the only known process to form a weak cusp in the center of galaxies. Therefore, our result very strongly suggests that luminous elliptical galaxies with weak cusps have experienced such "fall-in" of MBHs. As suggested by ME96, mergings of galaxies with MBHs already in the center is the most natural scenario for such an event. In other words, luminous ellipticals are most likely merger remnants. Our result also gives us an important suggestion for the origin of central MBHs, which are believed to exist in many galaxies (Richstone et al. 1998). The central MBHs in luminous elliptical galaxies with weak cusps were not formed there by some process such as gas accretion but were imported dynamically from the progenitor galaxies through the merging of them. If the central BH was formed in the timescale longer than the dynamical timescale, the central density cusp would have the slope of $-3/2$ or steeper (Bahcall & Wolf 1976; Young 1980). Therefore, the observed shallow cusp and the existence of central MBHs are consistent only if ellipticals with shallow cusps are merger remnants. The clear dichotomy of weak cusps and steep cusps, and the correlation between the slope of the cusp and the normalized rotation velocity v/σ (Figure 5) suggests that not only the merger of gas-poor ellipticals but also the "major merger" (Barnes & Hernquist 1992) of spirals (Barnes 1997) resulted mostly in weak cusps. Note that this clear separation is also visible in the sample of Carollo et al (1997, Figure 8). This connection between central slope and kinematics implies that in the merging of two spirals, BHs would have become massive before two galaxies finally merge, through gas-fueling induced by the tidal torque (Selwood & Moore 1999), and moreover, the rest of the gas would be ejected or formed into stars more rapidly than the BHs sink to the center of merger remnant.

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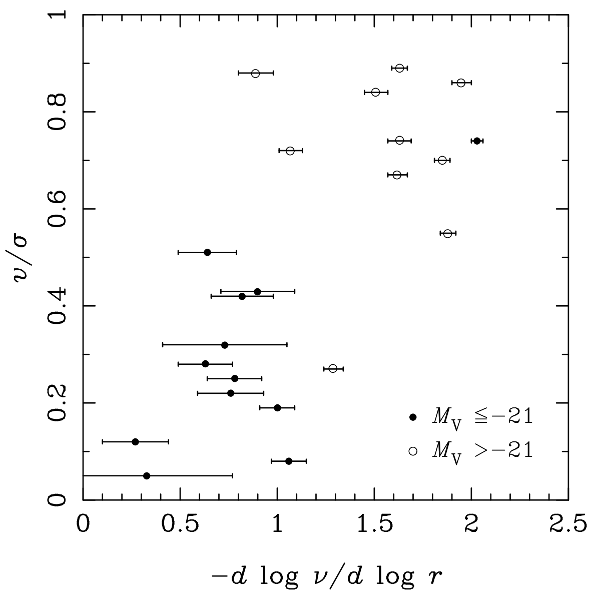

Fig. 5.— Normalized rotation velocity of galaxies v/σ versus cusp slope of luminosity density profile at $r = 0.1''$ obtained by *HST* observations (Gebhardt et al. 1996; Faber et al. 1997). Filled circles denote the galaxies brighter than $M_{\rm V} = -21$ and open circles denote those fainter than $M_{\rm V} = -21$.

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